

Statistics

Lecture 10



Feb 19-8:47 AM

Binomial Prob. Dist.:

(S6.16)

1) we have n independent events.

2) Each event has only two outcomes

$$P(\text{Success}) = p \quad P(\text{Failure}) = q$$

$$p + q = 1$$

p & q remain unchanged for all events.

3) $x \rightarrow \#$ Successes

$n-x \rightarrow \#$ Failure

$$P(x) = {}^nC_x \cdot p^x \cdot q^{n-x}$$

$${}^nC_x = \frac{n!}{x! \cdot (n-x)!}$$

it gives us the # of ways that different Combination can happen.

5 items, take 2 items, How many ways

$${}_5C_2 = \frac{5!}{2! \cdot (5-2)!}$$

$$= \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$

Can this happen?
order does not matter.

10

using TI:

5 [Math] [2nd] [PRB] [d] [nCr] 2 [Enter]

Jul 15-4:32 PM

12 players, Coach needs 5 to play.

How many different combination?

$$\begin{aligned}
 {}^{12}C_5 &= \frac{12!}{5! \cdot (12-5)!} \\
 &= \frac{12!}{5! \cdot 7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{7!}} \\
 &= 11 \cdot 9 \cdot 8 = \boxed{792}
 \end{aligned}$$

12 $\boxed{\text{Math}} \rightarrow \boxed{\text{PRB}} \downarrow \boxed{{}^nC_r} 5 \boxed{\text{enter}}$ 

CA Lotto, 50 numbers, choose 5

of Combination

$${}^{50}C_5 \quad \boxed{2,118,760}$$

50 $\boxed{\text{Math}} \rightarrow \boxed{\text{PRB}} \downarrow \boxed{{}^nC_r} 5 \boxed{\text{enter}}$

Jul 15-4:41 PM

Consider a binomial Prob. dist. with

$$n = 10 \text{ and } p = .4$$

 independent events

 Prob. of Success
Per event

$$1) q = 1 - p = 1 - .4 = \boxed{.6}$$

$$2) P(\text{exactly } 3 \text{ successes}) = P(x=3)$$

$$\text{Formula } {}^nC_x \cdot p^x \cdot q^{n-x}$$

$${}^{10}C_3 \cdot (.4)^3 \cdot (.6)^7 = \boxed{\rightarrow}$$

$$10 \quad \boxed{\text{Math}} \quad \boxed{\text{PRB}} \quad \boxed{{}^nC_r} \quad 3 \quad \times \quad .4 \quad \wedge \quad 3 \quad \times \quad .6 \quad \wedge \quad 7 \quad \boxed{\text{Enter}}$$

$$\approx \boxed{.215}$$

Jul 15-4:49 PM

I Slipped a fair coin 12 times.
Success is to land tails.

1) $n=12$ 2) $p=.5$ 3) $q=.5$

4) Compute $np=12(.5)=\boxed{6}$ 5) Compute $npq=12(.5)(.5)=\boxed{3}$

6) Compute $\sqrt{npq}=\sqrt{3}\approx 1.732$

7) $P(\text{exactly } 5 \text{ tails})=P(X=5)$

Formula $nC_x \cdot p^x \cdot q^{n-x}$
 $12C_5 \cdot (.5)^5 \cdot (.5)^7 \approx \boxed{.193}$

12 [Math] [PRB] [nCr] 5 [x] .5 [^] 5 [x] .5 [^] 7 [Enter]

Now using TI Command

[2nd] [VARS] [D] [binompdf] No Menu
 n, p, x

$\boxed{\text{binompdf}(12, .5, 5)}$ Trials: 12
 $P: .5$
 $x \text{ Value: } 5$

Your work [Paste] [Enter] $\boxed{.193}$

Jul 15-4:56 PM

Consider a binomial Prob. dist with
 $n=40$; $P=.8$

1) $q=1-p=1-.8=\boxed{.2}$

2) $np=40(.8)=\boxed{32}$ 3) $npq=40(.8)(.2)=\boxed{6.4}$ 4) $\sqrt{npq}=\sqrt{6.4}\approx\boxed{2.530}$

5) $P(\underline{x=35})=\text{binompdf}(40, .8, 35)=\boxed{.085}$
 $\uparrow$
 exactly 35 Successes

6) $P(\underline{x\leq 35})=P(X=35)+P(X=34)+P(X=33)+\dots+P(X=0)$
 at most 35 Successes
 $=\text{binomcdf}(40, .8, 35)=\boxed{.924}$

7) $P(\underline{x<35})=P(X\leq 34)$
 below less than
 $=\text{binomcdf}(40, .8, 34)=\boxed{.839}$

Jul 15-5:07 PM

I flip a loaded coin 125 times.

Success is to land tails.

Prob. of landing tails per flip is .6

$$1) n = 125 \quad 2) p = .6 \quad 3) q = .4$$

$$4) np = 125(.6) = 75 \quad 5) npq = 125(.6)(.4) = 30 \quad 6) \sqrt{npq} = \sqrt{30} \approx 5.477$$

7) $P(\text{land exactly 80 tails})$

$$= P(X = 80) = \text{binompdf}(125, .6, 80) = .049$$

8) $P(\text{land at most 80 tails}) = P(X \leq 80)$

$$= \text{binomcdf}(125, .6, 80) = .842$$

9) $P(\text{land at least 80 tails}) = P(X \geq 80)$

~~We don't want 79~~ We want 80 $= 1 - P(X \leq 79)$
 $= 1 - \text{binomcdf}(125, .6, 79)$
 $= .206$

Jul 15-5:20 PM

You are taking a True-False exam with 100 questions.

You are making random guesses.

Success is to get a correct ans.

$$1) n = 100 \quad 2) p = .5 \quad 3) q = .5$$

$$4) np = 100(.5) = 50 \quad 5) npq = 100(.5)(.5) = 25 \quad 6) \sqrt{npq} = \sqrt{25} = 5$$

7) $P(\text{guess exactly 60 correct answers})$

$$= P(X = 60) = \text{binompdf}(100, .5, 60) = .011$$

8) $P(\text{guess at most 60 correct answers})$

$$= P(X \leq 60) = \text{binomcdf}(100, .5, 60) = .982$$

9) $P(\text{guess more than 60 correct answers})$

$$P(X > 60) = P(X \geq 61) = 1 - P(X \leq 60)$$

~~We don't want 60~~ We want 61 $= 1 - .982$
 $= .018$

Jul 15-5:34 PM

Working with binomial Prob. dist.

$$\text{Mean } \mu = np$$

$$\text{Variance } \sigma^2 = npq$$

$$\text{Standard Deviation } \sigma = \sqrt{\sigma^2}$$

Consider a binomial Prob. dist with
 $n=150$, and $p=.6$

$$1) q = 1 - p = 1 - .6 = \boxed{.4}$$

$$2) \mu = np = 150(.6) = \boxed{90}$$

$$3) \sigma^2 = npq = 150(.6)(.4) = \boxed{36}$$

$$4) \sigma = \sqrt{\sigma^2} = \sqrt{36} = \boxed{6}$$

$$5) \text{ Usual Range} = \mu \pm 2\sigma = 90 \pm 2(6) \\ = 90 \pm 12 \\ \Rightarrow \boxed{78 \text{ to } 102}$$

Jul 15-5:59 PM

400 Voters were randomly selected.

Prob. that any voter is in favor of gun control is .8.

Success is being in favor of gun control.

$$1) n = 400 \quad 2) p = .8 \quad 3) q = .2$$

$$4) \mu = np = 400(.8) = \boxed{320}$$

$$5) \sigma^2 = npq = 400(.8)(.2) = \boxed{64}$$

$$6) \sigma = \sqrt{\sigma^2} = \sqrt{64} = \boxed{8}$$

$$7) 68\% \text{ Range} = \mu \pm \sigma = 320 \pm 8 \Rightarrow \boxed{312 \text{ to } 328}$$

$$8) 95\% \text{ Range} = \text{Usual Range} = \mu \pm 2\sigma \\ = 320 \pm 2(8) \\ = 320 \pm 16 \\ \Rightarrow \boxed{304 \text{ to } 336}$$

Jul 15-6:04 PM

9) $P(\text{\# of Voters in favor of is 300})$

$$P(X = 300) = \text{binomcdf}(400, .8, 300)$$

$$= \boxed{.002}$$

10) $P(\text{\# of voters in favor of is below 325})$

$$P(X < 325) = P(X \leq 324)$$

$$= \text{binomcdf}(400, .8, 324)$$

$$= \boxed{.710}$$

11) $P(\text{\# of voters in favor of is between 304 and 336 inclusive})$

$$P(304 \leq X \leq 336) = P(X \leq 336) - P(X \leq 303)$$

$$= \text{binomcdf}(400, .8, 336) -$$

$$\text{binomcdf}(400, .8, 303)$$

$$= \boxed{.961} \approx 96\%$$

Jul 15-6:12 PM

400 newborn babies are randomly selected.

Success is to have a boy.

1) $n = 400$

2) $p = .5$

3) $q = .5$

4) $\mu = np = 400(.5)$
 $= \boxed{200}$

5) $\sigma^2 = npq$
 $= 400(.5)(.5)$
 $= \boxed{100}$

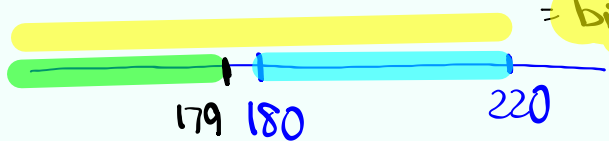
6) $\sigma = \sqrt{\sigma^2}$
 $= \sqrt{100}$
 $= \boxed{10}$

7) Usual Range = 95% Range = $\mu \pm 2\sigma$
 $= 200 \pm 2(10)$
 $= 200 \pm 20$
 $\Rightarrow \boxed{180 \text{ to } 220}$

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8) $P(\text{# of boys is between 180 and 220, inclusive})$

$$P(180 \leq X \leq 220) = P(X \leq 220) - P(X \leq 179)$$

$$= \text{binomcdf}(400, .5, 220) - \text{binomcdf}(400, .5, 179)$$


$$\approx \boxed{.960} \text{ } 96\%$$

Jul 15-6:27 PM

You are taking a multiple-choice exam with 60 questions.

You are making random guesses.

Success is to guess correctly

Each question has 5 choices but only one correct choice.

$$1) n = 60 \quad 2) p = \frac{1}{5} = .2 \quad 3) q = \frac{4}{5} = .8$$

$$4) \mu = np = 60(.2) = \boxed{12}$$

$$5) \sigma^2 = npq = 60(.2)(.8) = \boxed{9.6}$$

$$6) \sigma = \sqrt{\sigma^2} = \sqrt{9.6} = 3.098 \approx \boxed{3}$$

$$7) \text{ Usual Range } \mu \pm 2\sigma = 12 \pm 2(3) = 12 \pm 6 \Rightarrow \boxed{6 \text{ to } 18}$$

$$7) P(6 \leq X \leq 18) = P(X \leq 18) - P(X \leq 5)$$

keep subtract 1

$$= \text{binomcdf}(60, .2, 18) - \text{binomcdf}(60, .2, 5)$$

$$= \boxed{.966} \approx 96.6\%$$

SG 16
Last Page
use exact value of $P \approx 96$

SG 16 ✓

Jul 15-6:32 PM

500 tickets are sold for \$2 each.
one ticket is drawn.

The winning tkt gets a laptop worth \$1000.
Find expected value per ticket for **Fundraiser**.

Net	P(Net)
2 - 1000	$\frac{1}{500}$
2 - 0	$\frac{499}{500}$

Net $\rightarrow$ L1

P(Net) $\rightarrow$ L2

E.V. = $\mu = \bar{x}$

1-Var stats with
L1 & L2

Set-up for **buyers**

Net	P(Net)
1000 - 2	$\frac{1}{500}$
0 - 2	$\frac{499}{500}$

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10 nickels 2 dimes Select 2 Coins
No replacement

NN ND DN DD

10¢ Total 15¢ Total 20¢ Total

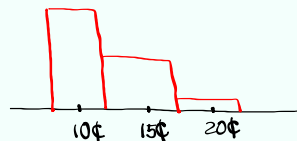
$$P(10¢ \text{ Total}) = P(NN) = \frac{10}{12} \cdot \frac{9}{11} = \frac{15}{22}$$

$$P(15¢ \text{ Total}) = P(ND \text{ or } DN) = 2 \cdot \frac{10}{12} \cdot \frac{2}{11} = \frac{10}{33}$$

$$P(20¢ \text{ Total}) = P(DD) = \frac{2}{12} \cdot \frac{1}{11} = \frac{1}{66}$$

15 $\div$ 22 $\div$ 10 $\div$ 33 $\div$ 1 $\div$ 66 $\div$ enter 1 ✓

T	P(T)
10¢	$\frac{15}{22}$
15¢	$\frac{10}{33}$
20¢	$\frac{1}{66}$



T $\rightarrow$ L1

P(T) $\rightarrow$ L2

1-Var stats with L1 & L2

$$\mu = \bar{x} = 11.6$$

$$n = 1 \checkmark$$

$$\sigma = \sigma_x \approx 2.513$$

$$\sigma^2 = \frac{625}{99}$$

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